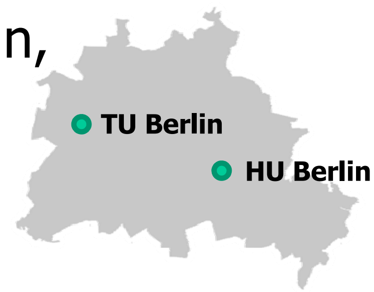
Algorithms and Data Structures

Marius Kloft

Who am I

Marius Kloft

2006	Diploma in Mathematics, U Marburg Minor: Computer Science
2007-2009	Doctoral Researcher, Fraunhofer & TU Berlin, *Machine Learning for Intrusion Detection*
2009-2010	Visiting Scholar, University of California
2010-2011	Doctorial Student, TU Berlin
2011	Dissertation on *Multiple Kernel Learning*
2011-2012	Postdoc, TU Berlin *ML for Genomics*
2012-2014	Postdoc, Courant Institute, Sloan-Kettering Cancer Center & Google Research
2014-	Junior Professor of Machine Learning (ML), HU Berlin



Lehrstuhl “Maschinelles Lernen”

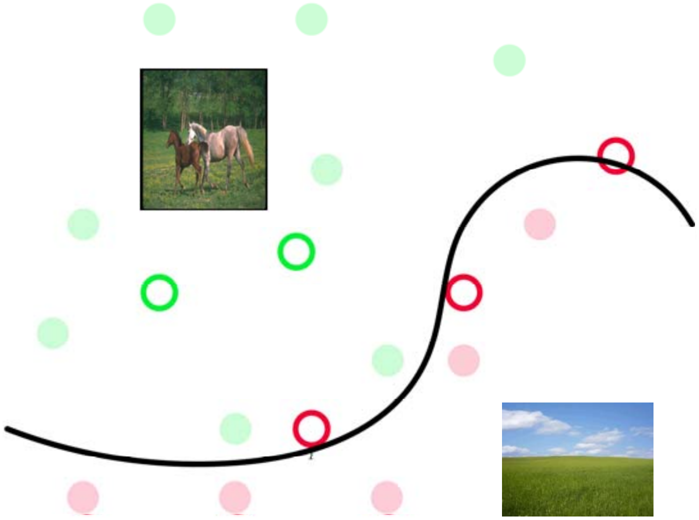
- Our topics in research
  - Development of novel machine learning algorithms
  - Speeding up machine learning algorithms to big data (e.g., via distributed computing)
  - Statistical learning theory
  - Applications in the biomedical domain

- Our topics in teaching
  - Machine Learning
  - Data Modeling
  - Algorithms & Data Structures

What is Machine Learning?

- Central question
  - "How to develop computer programs that learn from data to make accurate predictions?"

- Example
  - Image classification



Once upon a Time …

- IT company A develops software for insurance company B
  - Volume: ~4M Euros
- B not happy with delivered system; doesn't want to pay
- A and B call a referee to decide whether requirements were fulfilled or not
  - Volume: ~500K Euros
- Job of referee is to understand requirements (~60 pages) and specification (~300 pages), survey software and manuals, judge whether the contract was fulfilled or not

One Issue

This is hardly testable

- Requirement: „Allows for smooth operations in daily routine“

One Issue

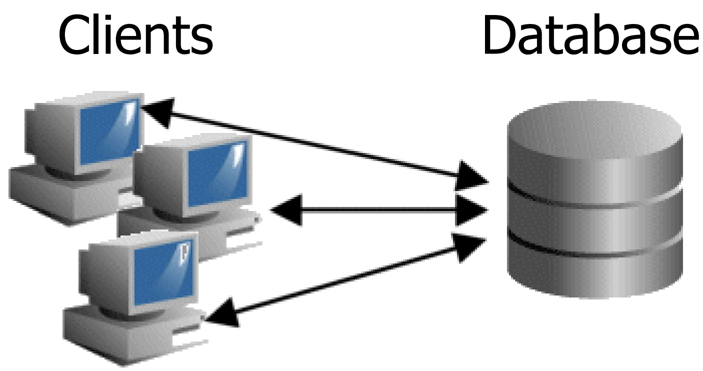
- Requirement: „Allows for smooth operations in daily routine“
- Claim from B
  - I search a specific contract
  - I select a region and a contract type
  - I get a list of all contracts sorted by name in a drop-down box
  - This sometimes takes minutes! A simple drop-down box! This performance is inacceptable for our call centre!

Discussion

- A: We tried and it worked fined
- B: OK, most of the times it works fine, but sometimes it is too slow
- A: We cannot reproduce the error; please be more specific in what you are doing before the problem occurs
- B: Come on, you cannot expect I log all my clicks and take notes on what is happening
- A: Then we conclude that there is no error
- B: Of course there is an error
- A: Please pay as there is no reproducible error
- …

A Closer Look

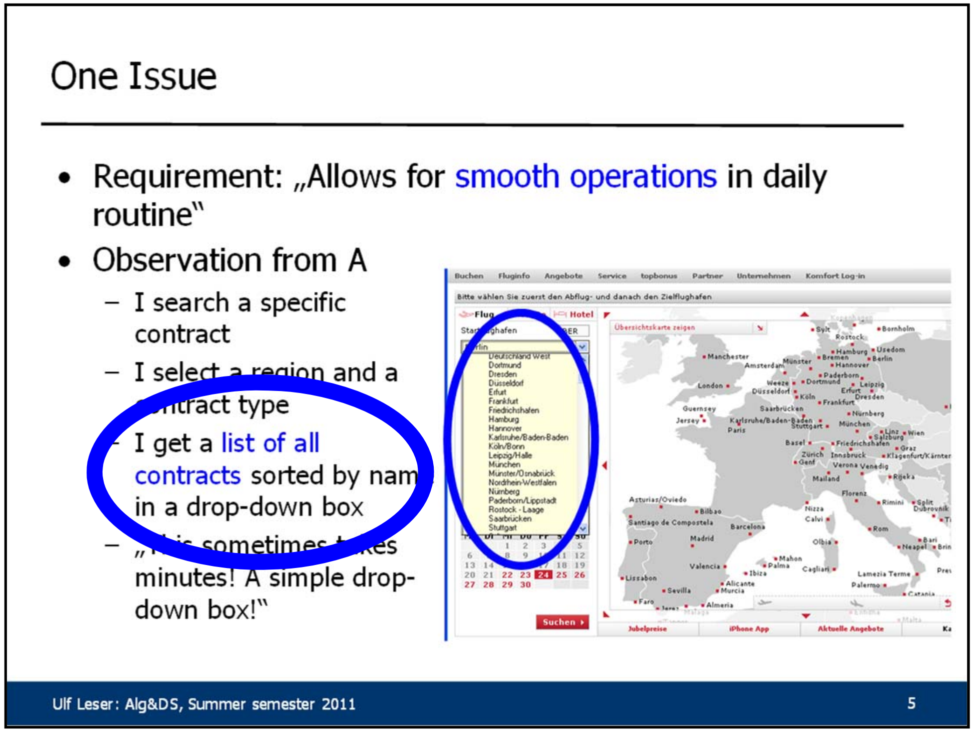
- System has classical two-tier architecture



- Upon selecting a region and a contract, a query is constructed and send to the database
- Procedure for “query construction” is used a lot
  - All contracts in a region, … running out this year, … by first letter of customer, … sum of all contract revenues per year, …
  - “Meta” coding: very complex, hard to understand

Requirement

- Recall



- After retrieving the list of customers, it has to be sorted

Code used for Sorting the List of Customer Names

```
S: array_of_names;
n := |S|;
for i = 1..n-1 do
  for j = i+1..n do
    if S[i]>S[j] then
      tmp := S[i];
      S[i] := S[j];
      S[j] := tmp;
    end if;
  end for;
end for;
```

- S: array of Strings, |S|=n
- Sort S alphabetically
  - Take the first string and compare to all others
  - Swap whenever a later string is smaller
  - Repeat for 2nd, 3rd, …
  - After 1st iteration of outer loop: S[1] contains smallest string from S
  - After 2nd iteration of outer loop: S[2] contains 2nd smallest string from S
  - etc.

Example

```
S: array_of_names;
n := |S|;
for i = 1..n-1 do
  for j = i+1..n do
    if S[i]>S[j] then
      tmp := S[i];
      S[i] := S[j];
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    end if;
  end for;
end for;
```
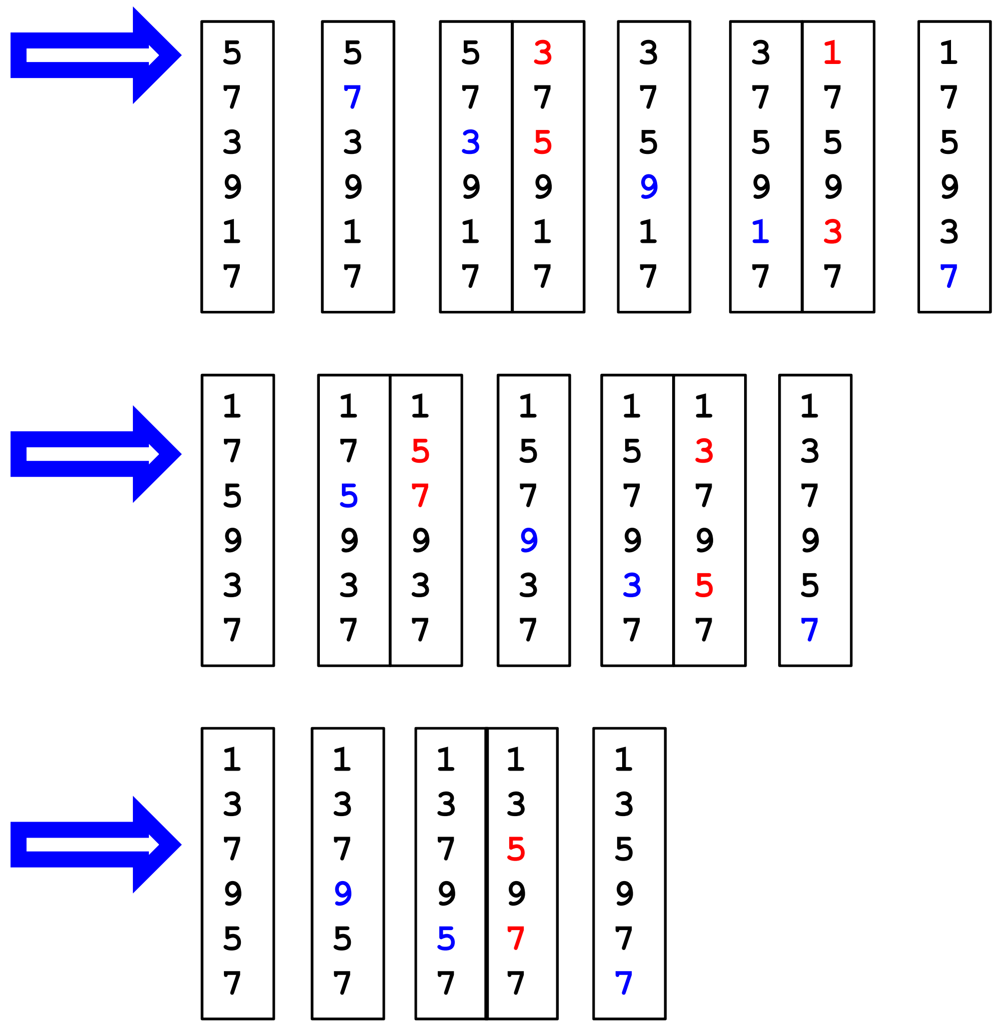
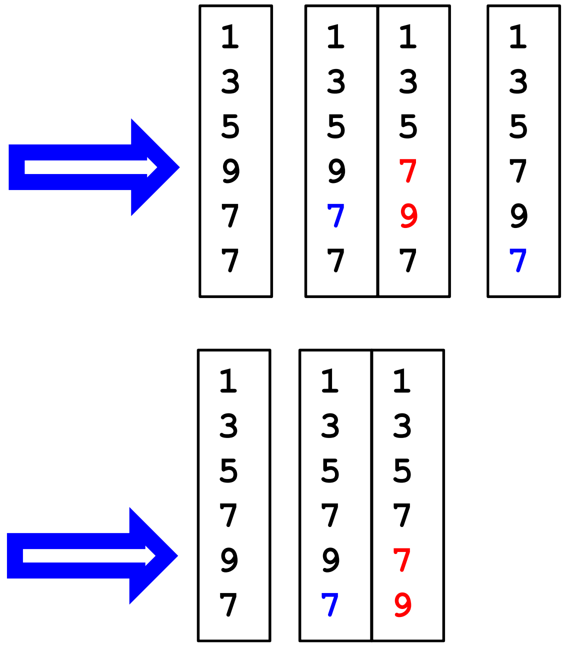



Example continued



- Seems to work
- This algorithm is called "selection sort"
  - Select smallest element and move to front, select second-smallest and move to 2nd position, …

Analysis

- How long will it take (depending on n)?
- Which parts of the program take CPU time?

1. Very little, constant time
2. Probably very little, constant time
3. n-1 assignments
    4. n-i assignments
        5. One comparison
            6. One assignment
            7. One assignment
            8. One assignment
        9. No time
    10. One increment (j+1); one test
11. One increment (i+1); one test

```
1.  S: array_of_names;
2.  n := |S|;
3.  for i = 1..n-1 do
4.    for j = i+1..n do
5.      if S[i]>S[j] then
6.        tmp := S[i];
7.        S[i] := S[j];
8.        S[j] := tmp;
9.      end if;
10.   end for;
11. end for;
```

Slightly More Abstract

- Assume one assignment/test costs c, one addition d
- Which parts of the program take time?

1. 0
2. c
3. (n-1)*c
    4. (n-i)*c (hmmm …)
        5. c
            6. c
            7. c
            8. c
        9. 0
    10. c+d
11. c+d

```
1.  S: array_of_names;
2.  n := |S|;
3.  for i = 1..n-1 do
4.    for j = i+1..n do
5.      if S[i]>S[j] then
6.        tmp := S[i];
7.        S[i] := S[j];
8.        S[j] := tmp
9.      end if;
10.   end for;
11. end for;
```

Slightly More Compact

- Assume one assignment/test costs c, one addition d
- Which parts of the program take time?
  - Let's be pessimistic: We always swap
    - How would the list have to look like in first place?
  - c
  - (n-1)*c* (
    - n-i* (
      - 5*c
    - c+d) +
  - c+d)

```
S: array_of_names;
n := |S|;
for i = 1..n-1 do
  for j = i+1..n do
    if S[i]>S[j] then
      tmp := S[i];
      S[i] := S[j];
      S[j] := tmp;
    end if;
  end for;
end for;
```

This is not yet clear

Even More Compact

- Assume one assignment/test costs c, one addition d
- Which parts of the program take time?
  - We have some cost outside the loop (out_loops)
  - And some cost inside the loop (in_loops)
  - How often do we need to perform in_loops?
  - c+(n-1)*c* ((n-i)*…)= out_loops+(n-1)*c*?*in_loops

out_loops (lines: S: array_of_names; n := |S|;)

in_loops (lines: if S[i]>S[j] then … end if;)

```
S: array_of_names;
n := |S|;
for i = 1..n-1 do
  for j = i+1..n do
    if S[i]>S[j] then
      tmp := S[i];
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      S[j] := tmp;
    end if;
  end for;
end for;
```
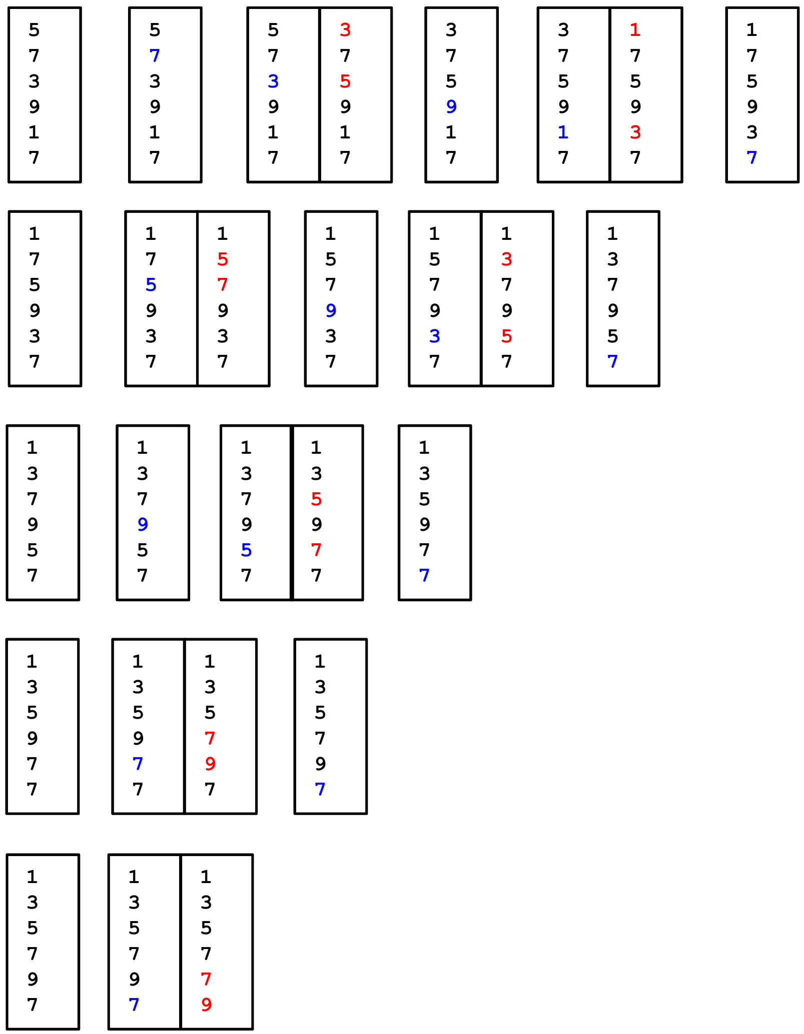
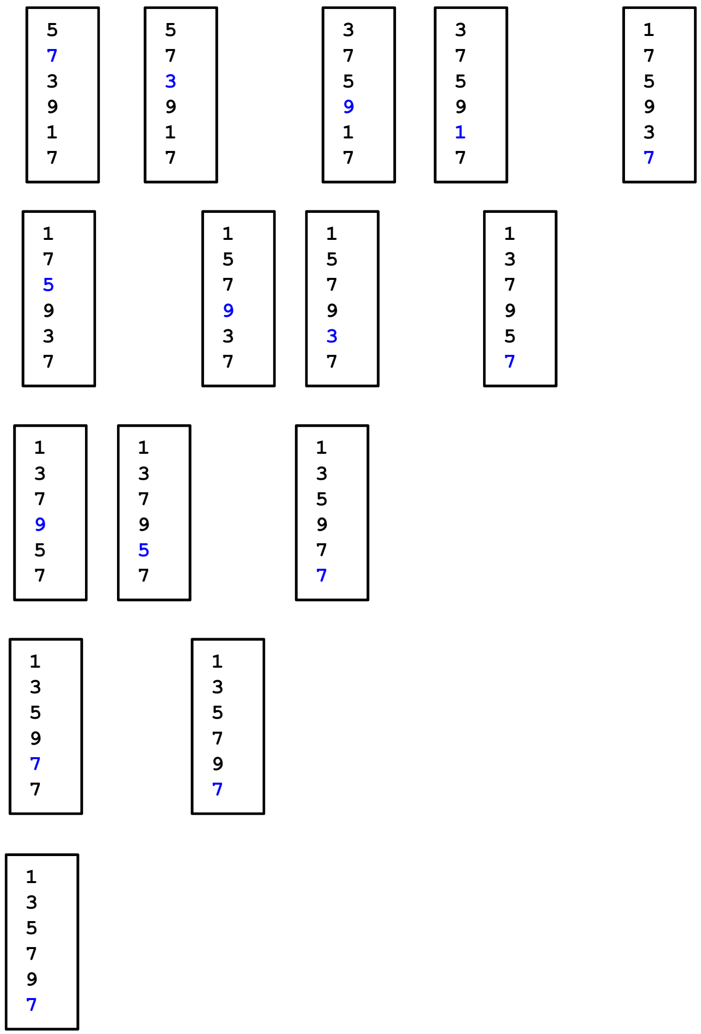

Observations



- The number of comparisons is independent of the number of swaps
  - We always compare, but we do not always swap

Observations



- The number of comparisons is independent of the number of swaps
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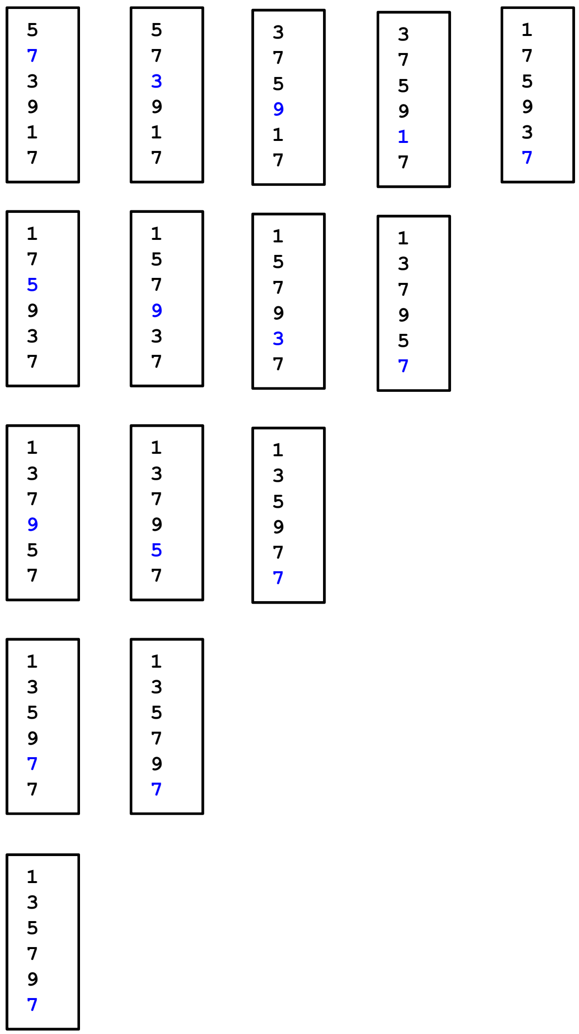
- How many comparisons do we perform in total?

Observations



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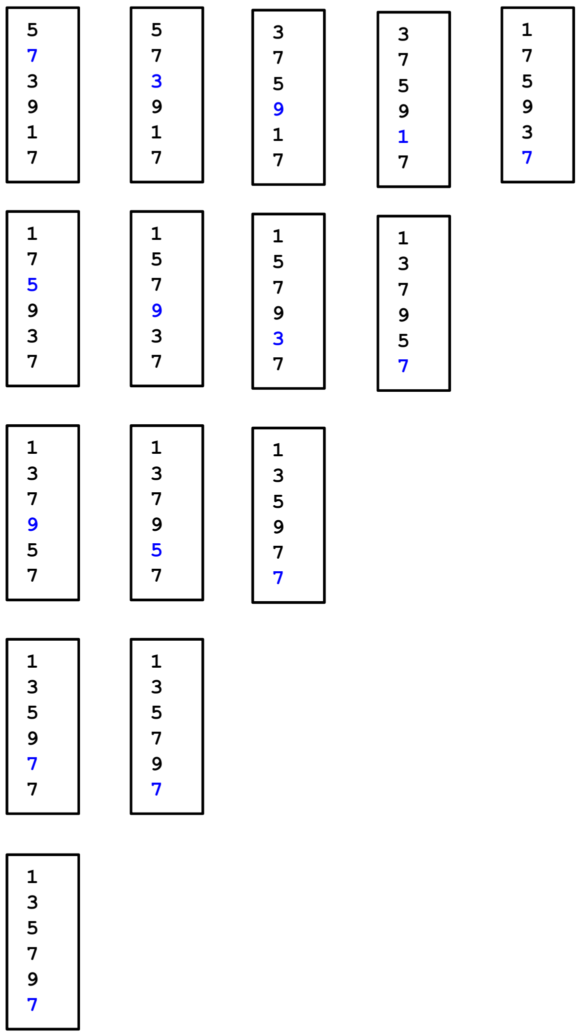


- First string is compared to n-1 other strings
  - First row
- Second is compared to n-2
  - Second row
- Third is compared to n-3
- …
- n-1'th is compared to 1

Together

$$(n-1)+(n-2)+(n-3)+\ldots+1=\sum_{i=1}^{n-1} i=\frac{n(n-1)}{2}=\frac{n^2}{2}-\frac{n}{2}$$

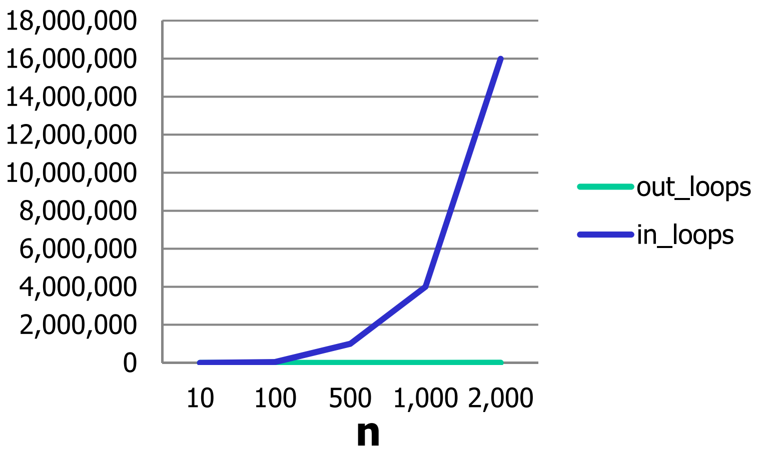
- This leads to the following total cost estimation:

  $out\_loops+(n^2-n)*in\_loops/2$

- Let's assume c=d=1, then:

  $n+1+(n^2-n)*8/2$

n	out_loops	in_loops	total
10	11	360	371
100	11	39.600	39.611
500	11	998.000	998.011
1.000	11	3.996.000	3.996.011
2.000	11	15.992.000	15.992.011

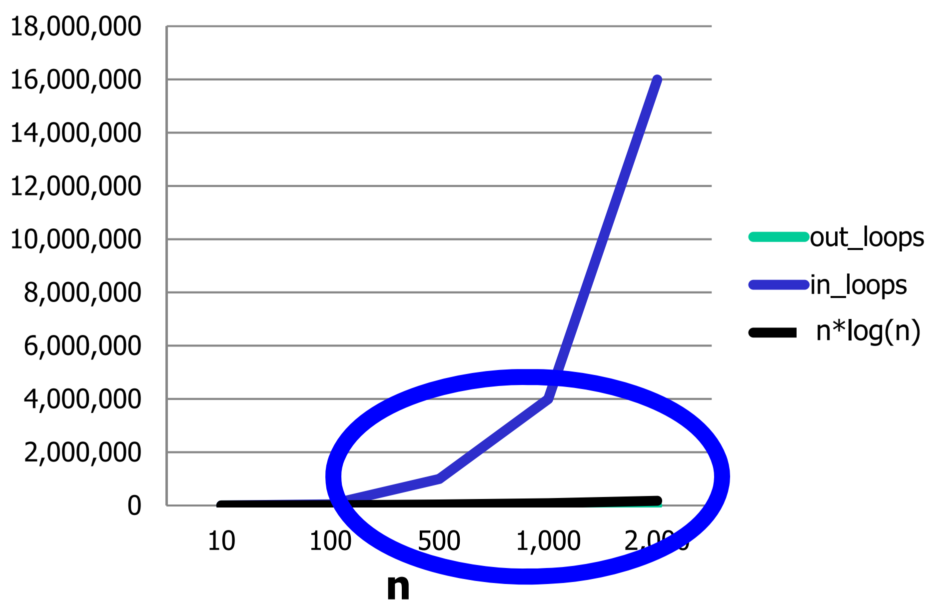


What Happened?

- Most combinations (region, contract type) select only a handful of contracts
- A few combinations select many contracts (2000-5000)
- Time it takes to fill the drop-down list is not proportional to the number of contracts (n), but proportional to $n^2/2$
  - Required time is "quadratic in n"
  - Assume one comparison takes 10 nanoseconds (0.000001 sec)
  - A handful of contracts (~10): ~500 operations => 0,0005 sec
  - Many contracts (~5000) => ~125M operations => 125 sec
  - Humans always expect linear time …
- Question: Could they have done it better?

Of course

- Efficient sorting algorithms need ~n*log(n)*x operations
  - Quick sort, merge sort, … see later
  - For comparability, let's assume x=8
  - Under certain reasonable assumptions, one cannot sort faster than with ~n*log(n) operations



"Almost" linear

So there is an End to Research in Sorting?

- We didn�one consider how long it takes to compare 2 strings
  - We used c=d=1, but we need to compare strings char-by-char
  - Time of every comparison is proportional to the length of the shorter string
- We want methods requiring less operations per inner loop
- We want algorithms that are fast even if we want to sort 1.000.000.000 strings
  - Which might not fit into main memory
- We made a pessimistic estimate – what is a realistic estimate (how often do we swap in the inner loop?)?
- …

Terasort Benchmark

- 2009: 100 TB in 173 minutes
  - Amounts to 0.578 TB/min
  - 3452 nodes x (2 Quadcore, 8 GB memory)
  - Owen O'Malley and Arun Murthy, Yahoo Inc.
- 2010: 1,000,000,000,000 records in 10,318 seconds
  - Amounts to 0.582 TB/min
  - 47 nodes x (2 Quadcore, 24 GB memory), Nexus 5020 switch
  - Rasmussen, Mysore, Madhyastha, Conley, Porter, Vahdat, Pucher
- Other goals
  - PennySort: Amount of data sorted for a penny's worth of system time
  - JouleSort: Minimize amount of energy required during sorting

Content of this Lecture

- This lecture
- Algorithms and …
- Data Structures
- Concluding Remarks

Algorithms and Data Structures

- Slides are English
- Vorlesung wird auf Deutsch gehalten
- Acknowledgement: Prof. Ulf Leser
- Lecture: 4 SWS; exercises 2 SWS
- Contact:
  - Marius Kloft
  - RUD 25, Raum 4.215
  - Office hours: Fridays, 15:00-16:00
  - "Email": only via Goya
  - Always cc your TA (=Übungsleiter(in)) when you write me a message

Exercises & TAs:

- Monday, 9-11, RUD 26, 1‘303, Marc Bux
- Monday, 13-15, RUD 26, 1‘305, Marc Bux
- Monday, 13-15, RUD 26, 1‘303, Florian Tschorsch
- Tuesday, 9-11, RUD 26, 1'303, Patrick Schäfer
- Tuesday, 13-15, RUD 26, 0‘313, Kim Völlinger
- Wednesday, 9-11, RUD 26, 1‘306, Berit Grußien
- Thursday, 13-15, RUD 26, 1‘305, Kim Völlinger
- Thursday, 13-15, RUD 26, 0'313, Patrick Schäfer
- Friday, 9-11, RUD 26, 1‘305, Berit Grußien
- Friday, 11-13, RUD 26, 1‘305, Florian Tschorsch

Schedule

- Tutorial: Michael R. Jung
  - Mondays, 17-19, RUD 26, 1‘303
  - Wednesdays, 17-19, RUD 26, 1‘303
  - Thursdays, 15-17, RUD 26, 1‘306
  - Fridays, 11-13, RUD 25, 3.101
- Mathematics refresher course:
  - Wednesday, 9-11, RUD 26, 1‘306, Berit Grußien
  - Thursday, 13-15, RUD 26, 0'313, Berit Grußien
- Exam:
  - Aug 1, 9:30-12:00, RUD 26, 0‘115 & 0‘110
    - „Klausureinsicht“: Aug 4, 11-13, RUD25, 3.101 & 3.113
  - Oct 4, 9:30-12:00, RUD 26, 0‘115 (Wiederholungsklausur)

Lecture

- Mondays & Wednesdays, RUD 26, 0'115
  - **11:00-11:45 & 12:00-12:45**
- We will make 15mins break

Exercises

- You will build teams of usually two students (maximally three) students registered in GOYA
- There will six bi-weekly assignments in total
- Each assignment gives 50 points
- Only groups having ≥50% of the maximal number of points over the entire semester are admitted to the exam

Exercises (continued)

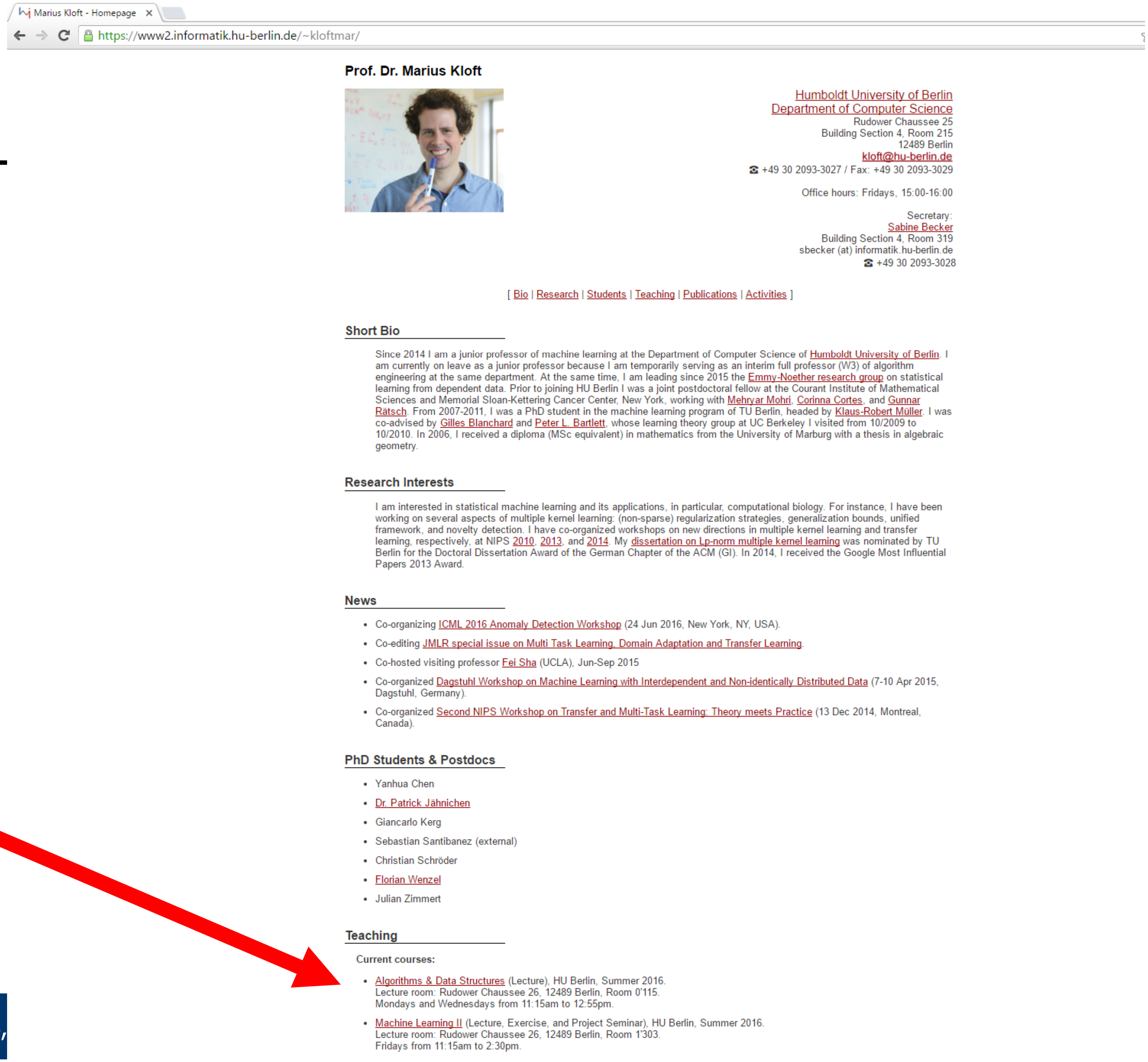
- Text-based homework assignments to be submitted in paper until 10:55 before the Monday lecture
  - Or earlier in the letterbox at RUD 25, 3.321
  - New problem sheet available on the same day
- One-time exception: this week‘s problem sheet will be released on Wednesday, April 20
  - You have time for submission until Wednesday, May 4
- First assignment available on Wednesday (is due May 2)
- Programming assignments to be tested with Java 1.6 on gruenau2 and submitted in GOYA (same deadline)

Literature

- Ottmann, Widmayer: Algorithmen und Datenstrukturen, Spektrum Verlag, 2002-2012
  - 20 copies in library

- Other
  - Saake / Sattler: Algorithmen und Datenstrukturen (mit Java), dpunkt.Verlag, 2006
  - Sedgewick: Algorithmen in Java: Teil 1 - 4, Pearson Studium, 2003
    - 20 copies in library
  - Güting, Dieker: Datenstrukturen und Algorithmen, Teubner, 2004
  - Cormen, Leiserson, Rivest, Stein: Introduction to Algorithms, MIT Press, 2003
    - 10 copies in library

Web



Marius Kloft - Homepage

https://www2.informatik.hu-berlin.de/~kloftmar/

Prof. Dr. Marius Kloft

Humboldt University of Berlin
Department of Computer Science
Rudower Chaussee 25
Building Section 4, Room 215
12489 Berlin
kloft@hu-berlin.de
☎ +49 30 2093-3027 / Fax: +49 30 2093-3029

Office hours: Fridays, 15:00-16:00

Secretary:
Sabine Becker
Building Section 4, Room 319
sbecker (at) informatik.hu-berlin.de
☎ +49 30 2093-3028

[Bio | Research | Students | Teaching | Publications | Activities]

Short Bio

Since 2014 I am a junior professor of machine learning at the Department of Computer Science of Humboldt University of Berlin. I am currently on leave as a junior professor because I am temporarily serving as an interim full professor (W3) of algorithm engineering at the same department. At the same time, I am leading since 2015 the Emmy-Noether research group on statistical learning from dependent data. Prior to joining HU Berlin I was a joint postdoctoral fellow at the Courant Institute of Mathematical Sciences and Memorial Sloan-Kettering Cancer Center, New York, working with Mehryar Mohri, Corinna Cortes, and Gunnar Rätsch. From 2007-2011, I was a PhD student in the machine learning program of TU Berlin, headed by Klaus-Robert Müller. I was co-advised by Gilles Blanchard and Peter L. Bartlett, whose learning theory group at UC Berkeley I visited from 10/2009 to 10/2010. In 2006, I received a diploma (MSc equivalent) in mathematics from the University of Marburg with a thesis in algebraic geometry.

Research Interests

I am interested in statistical machine learning and its applications, in particular, computational biology. For instance, I have been working on several aspects of multiple kernel learning: (non-sparse) regularization strategies, generalization bounds, unified framework, and novelty detection. I have co-organized workshops on new directions in multiple kernel learning and transfer learning, respectively, at NIPS 2010, 2013, and 2014. My dissertation on Lp-norm multiple kernel learning was nominated by TU Berlin for the Doctoral Dissertation Award of the German Chapter of the ACM (GI). In 2014, I received the Google Most Influential Papers 2013 Award.

News

- Co-organizing ICML 2016 Anomaly Detection Workshop (24 Jun 2016, New York, NY, USA).
- Co-editing JMLR special issue on Multi Task Learning, Domain Adaptation and Transfer Learning.
- Co-hosted visiting professor Fei Sha (UCLA), Jun-Sep 2015
- Co-organized Dagstuhl Workshop on Machine Learning with Interdependent and Non-identically Distributed Data (7-10 Apr 2015, Dagstuhl, Germany).
- Co-organized Second NIPS Workshop on Transfer and Multi-Task Learning: Theory meets Practice (13 Dec 2014, Montreal, Canada).

PhD Students & Postdocs

- Yanhua Chen
- Dr. Patrick Jähnichen
- Giancarlo Kerg
- Sebastian Santibanez (external)
- Christian Schröder
- Florian Wenzel
- Julian Zimmert

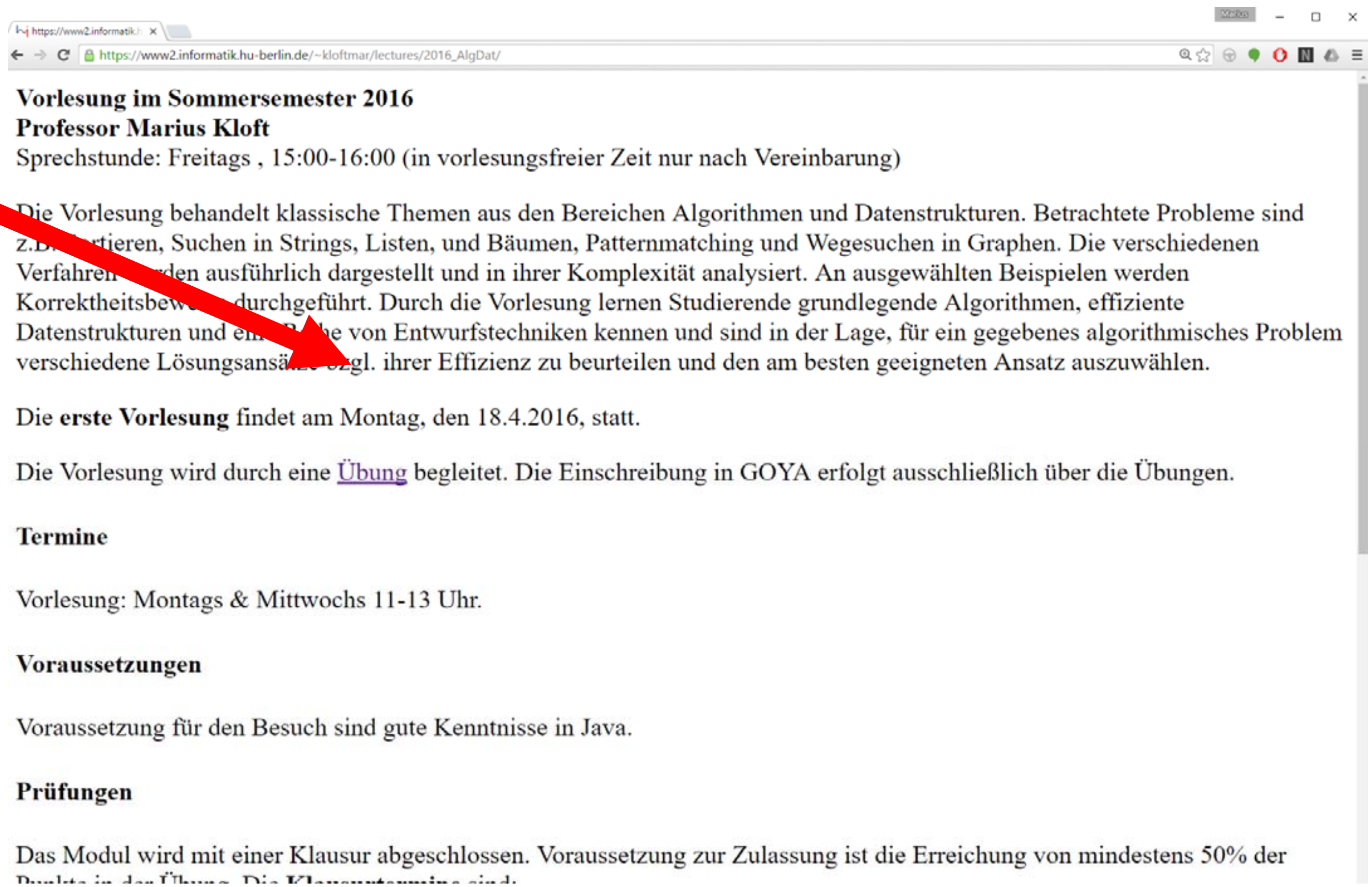
Teaching

Current courses:

- Algorithms & Data Structures (Lecture), HU Berlin, Summer 2016.
  Lecture room: Rudower Chaussee 26, 12489 Berlin, Room 0'115.
  Mondays and Wednesdays from 11:15am to 12:55pm.
- Machine Learning II (Lecture, Exercise, and Project Seminar), HU Berlin, Summer 2016.
  Lecture room: Rudower Chaussee 26, 12489 Berlin, Room 1'303.
  Fridays from 11:15am to 2:30pm.

Website: Lecture

- https://hu.berlin/vl_algodat16



Vorlesung im Sommersemester 2016
Professor Marius Kloft
Sprechstunde: Freitags , 15:00-16:00 (in vorlesungsfreier Zeit nur nach Vereinbarung)

Die Vorlesung behandelt klassische Themen aus den Bereichen Algorithmen und Datenstrukturen. Betrachtete Probleme sind z.B. Sortieren, Suchen in Strings, Listen, und Bäumen, Patternmatching und Wegesuchen in Graphen. Die verschiedenen Verfahren werden ausführlich dargestellt und in ihrer Komplexität analysiert. An ausgewählten Beispielen werden Korrektheitsbeweise durchgeführt. Durch die Vorlesung lernen Studierende grundlegende Algorithmen, effiziente Datenstrukturen und eine Reihe von Entwurfstechniken kennen und sind in der Lage, für ein gegebenes algorithmisches Problem verschiedene Lösungsansätze bzgl. ihrer Effizienz zu beurteilen und den am besten geeigneten Ansatz auszuwählen.

Die **erste Vorlesung** findet am Montag, den 18.4.2016, statt.

Die Vorlesung wird durch eine Übung begleitet. Die Einschreibung in GOYA erfolgt ausschließlich über die Übungen.

Termine

Vorlesung: Montags & Mittwochs 11-13 Uhr.

Voraussetzungen

Voraussetzung für den Besuch sind gute Kenntnisse in Java.

Prüfungen

Das Modul wird mit einer Klausur abgeschlossen. Voraussetzung zur Zulassung ist die Erreichung von mindestens 50% der Punkte in der Übung. Die Klausurtermine sind:

Website: Excercises

Übung Algorithmen und [...]

https://www.informatik.hu-berlin.de/de/forschung/gebiete/wbi/teaching/archive/ss16/algodat16

Archiv

- SS 16
  - Vorlesung Informationsintegration
  - Übung Informationsintegration
  - Vorlesung Grundlagen der Bioinformatik
  - Übung Grundlagen der Bioinformatik
  - Proseminar Wissenschaftliches Arbeiten
  - **Übung Algorithmen und Datenstrukturen**
- WS 15/16
- SS15
- WS 14/15
- SS14
- WS 13/14
- SS13
- WS 12/13
- SS12
- WS 11/12
- SS 11

Übung Algorithmen und Datenstrukturen

Informationen und Materialien zur begleitenden Übung der Vorlesung Algorithmen und Datenstrukturen im Sommersemester 2016

Bearbeitung und Abgabe

Erster Übungstermin: 25.04.2016

Das erste Übungsblatt wird am Mittwoch, den 20.04. auf dieser Seite und in GOYA veröffentlicht. Die Abgabe des ersten Übungsblatts erfolgt dann am 04.05. Anschließend werden die Übungsblätter alle zwei Wochen montags online gestellt, beginnend mit dem zweiten Übungsblatt am 02.05. Die Abgabe erfolgt von da an jeweils am übernächsten Montag. Sie haben also immer zwei Wochen Bearbeitungszeit für jedes Aufgabenblatt, wobei sich die ersten beiden Übungsblätter um zwei Tage überschneiden. Jedes Übungsblatt hat vier Aufgaben, für die insgesamt 50 Punkte erhältlich sind. Insgesamt wird es sechs Übungsblätter geben.

Die Lösungen sind auf nach Aufgaben getrennten Blättern abzugeben. Heften Sie bitte die zu einer Aufgabe gehörenden Blätter vor der Abgabe zusammen. Der schriftliche Teil der Lösungen ist Montag bis TBA Uhr **vor** der Vorlesung im Hörsaal oder bis 10:45 Uhr im Briefkasten (RUD 25, Raum 3.321) abzugeben. Den elektronischen Teil reichen Sie über GOYA ein.

Die Übungsaufgaben sind in Gruppen von je zwei, in Ausnahmefällen auch drei, Studenten zu bearbeiten. Studenten die nach der ersten Woche noch keinen Übungspartner haben, bekommen einen Übungspartner zugewiesen.

Die Implementationsaufgaben sind in Java zu lösen (Version 1.6 oder niedriger). Als Referenzrechner können Sie den Instituts-Rechner gruenau2 verwenden. Bitte stellen Sie sicher, dass Ihre Abgabe auf diesem Rechner kompiliert und läuft. **Abgaben die sich nicht kompilieren und ausführen lassen werden mit 0**

Pseudo Code

- You need to program all exercises in Java
- I will use informal pseudo code
  - Much more concise than Java
  - Goal: You should understand what I mean
  - Syntax is not important; don’t try to execute programs from slides
- Translation into Java should be simple

Topics of the Course

- Intro (~2)
- Complexity (~1)
- Abstract data types (~2)
- Lists (~2)
- Sorting (~3)
- Searching in lists (~4)
- Queues & Hashing (~3)
- Search trees (~4)
- Graphs (~5)
- The end (~1)

April

Mai

June

July

Questions?

Questions

- BSc CS?
- Diplom CS?
- BSc Mathematics?
- Kombibachelor?
- INFOMIT? Biophysics? Beifach?
- Semester?
- Who heard this course before?

Content of this Lecture

- This lecture
- Algorithms and …
- Data Structures
- Concluding Remarks

What is an Algorithm?

- An algorithm is a recipe for doing something
  - Washing a car, sorting a set of strings, preparing a pancake, employing a student, …
- The recipe is given in a (formal, clearly defined) language
- The recipe consists of atomic steps
  - Someone (the machine) must know what to do
- The recipe must be precise
  - After every step, it must be uniquely decidable what comes next
  - Does not imply that every run has the same sequence of steps
- The recipe must not be infinitely long

More Formal

- Definition (general)
  An algorithm is a precise and finite description of a process consisting of elementary steps.
- Definition (Computer Science)
  An algorithm is a precise and finite description of a process that is (a) given in a formal language and (b) consists of elementary and machine-executable steps.

- Usually we also want: “and (c) solves a given problem”
  - But algorithms can be wrong …

Almost Synonyms

- Rezept
- Ausführungsvorschrift
- Prozessbeschreibung
- Verwaltungsanweisung
- Regelwerk
- Bedienungsanleitung
  - Well …
- …

History

- Word presumably dates back to "Muhammed ibn Musa abu Djafar alChoresmi",
  - Published a book on calculating in the 8th century in Persia
  - See Wikipedia for details
- Given the general meaning of the term, there have been algorithms since ever
- One of the first prominent one in math: Euclidian algorithm for finding the greatest common divisor (gcd) of two ints
  - Assume $a,b \geq 0$; define $gcd(a,0)=a$

Euclidian Algorithm

Actually not really precise

- Recipe: Given two integers a, b. As long as neither a nor b is 0, take the smaller of both and subtract it from the greater. If this yields 0, return the other number
- Example: (28, 92)
  - (28, 64)
  - (28, 36)
  - (28, 8)
  - (20, 8)
  - (12, 8)
  - (4, 8)
  - (4, 4)
  - (4, 0)

```
a,b: integer;
if a=0 return b;
while b≠0
  if a>b
    a := a-b;
  else
    b := b-a;
  end if;
end while;
return a;
```

- Will this always work?

Proof (sketch) that an Algorithm is Correct

```
func euclid(a,b: int)
  if a=0 return b;
  while b≠0
    if a>b
      a := a-b;
    else
      b := b-a;
    end if;
  end while;
  return a;
end func;
```

- Assume our function "euclid" returns x
- We write "b|a" if (a mod b)=0
  - We say: "b teilt a"
- 1st step: x is a common divisor of a and b
  - Last step: b=0 and x=a≠0 $\Rightarrow$ x|a, x|b
  - Pre-last: It must hold: a=b $\Rightarrow$ x|a, x|b
  - Previous: Either a=2x or b=2x $\Rightarrow$ x|a, x|b
  - Previous: Either (a,b)=(3x,x) or (a,b)=(2x,3x) or (a,b)=(x,3x) or (a,b)=(3x,2x) $\Rightarrow$ x|a, x|b
  - …

Proof (sketch) that an Algorithm is Correct

```
func euclid(a,b: int)
  if a=0 return b;
  while b≠0
    if a>b
      a := a-b;
    else
      b := b-a;
    end if;
  end while;
  return a;
end func;
```

- Note: if $c|a$ and $c|b$ and $a>b \Rightarrow c|(a-b)$
- 2nd step: x is the greatest divisor
  - Assume some y with $y|a$ and $y|b$
  - It follows that $y|(a-b)$ (or $y|(b-a)$)
  - It follows that $y|((a-b)-b)$ (or $y|((b-a)-b)$ …)
  - …
  - It follows that $y|x$
  - Thus, $y \le x$

Properties of Algorithms

- Definition
  An algorithm is called terminating if it stops after a finite number of steps for every valid input
- Definition
  An algorithm is called deterministic if it always performs the same series of steps given the same input

- We only study terminating and mostly deterministic algs
  - Operating systems are “algorithms” that do not terminate
  - Algs randomly deciding about next steps are not deterministic

Algorithms and Runtimes

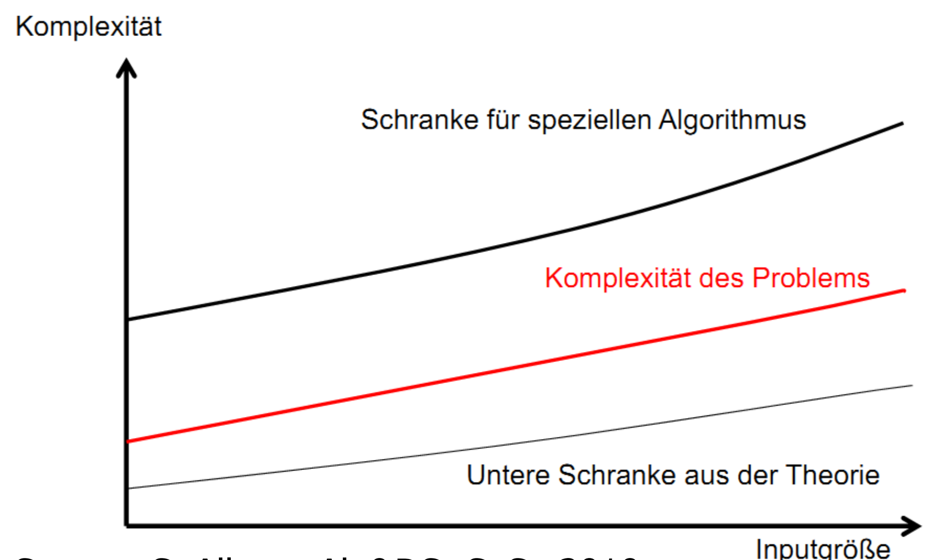
- Usually, one seeks efficient (read: fast) algorithms
- We will analyze the efficiency of an algorithm as a function of the size of its input; this is called its (time-)complexity
  - Selection-sort has time-complexity "$O(n^2)$"
- The real runtime of an algorithm on a real machine depends on many additional factors we gracefully ignore
  - Clock rate, processor, programming language, representation of primitive data types, available main memory, cache lines, …
- But: Complexity in some sense correlates with runtime
  - It should correlate well in most cases, but there may be exceptions (especially on small inputs)

Algorithms, Complexity and Problems

- An (correct) algorithm solves a given problem
- An algorithm has a certain complexity
  - Which is a statement about the time it will take to finish as a function on the size of its input
- Also problems have complexities
  - The complexity of a problem is a lower bound on the complexity of any algorithm that solves it
  - If an algorithm has the same complexity as the problem it solves, it is optimal – no algorithm can solve this problem faster
- Proving the complexity of a problem usually is much harder than proving the complexity of an algorithm
  - Needs to make a statement about any possible algorithm

Relationships

- There are problems for which we know their complexity, but no optimal algorithm is known
- There are problems for which we do not know the complexity yet more and more efficient algorithms are discovered over time
- There are problems for which we only know lower thresholds on their complexity, but not the precise complexity
- There are problems of which we know that no algorithm exists
  - Undecidable problems
  - Example: "Halteproblem"
  - Implies that we cannot check in general if an algorithm is terminating



Source: S. Albers, Alg&DS; SoSe 2010

Properties of Algorithms

1. Efficiency – how long will it take?
   - Time complexity
   - Worst-case, average-case, best-case
   - Alternative: Run on reference machine usi
     - Done a lot in practical algorithm engineeri
     - Not so much in this introductory course
2. Space consumption – how much memory will it need?
   - Space complexity
   - Worst-case, average-case, best-case
   - Can be decisive for large inputs
3. Correctness – does the algorithm solve the problem?

Often, one can trade space for time – look at both

In This Module

- We will only occasionally look at space complexity
- We will mostly focus on worst-case time complexity
  - Best-case is not very interesting
  - Average-case often is hard to determine
    - What is an „average string list"?
    - What is the average length of an arbitrary string?
    - May depend in the semantic of the input (person names, DNA sequences, job descriptions, book titles, language, …)
- Keep in mind: Worst-case often is overly pessimistic

Content of this Lecture

- This lecture
- Algorithms and …
- Data Structures
- Concluding Remarks

What is a Data Structure?

- Algorithms work on input data, generate intermediate data, and finally produce result data
- A data structure is a way how data is represented inside the machine
  - In memory or on disc (see Database course)
- Data structures determine what algs may do at what cost
  - More precisely: … what a specific step of an algorithm costs
- Complexity of algs is tightly bound to the ds they use
  - So tightly that one often subsumes both concepts under the term "algorithm"

Example: Selection Sort (again)

- We assumed that S is
  - a list of strings (abstract), represented
  - as an array (concrete data structure)
- Arrays allow us to access the i'th element with a cost that is independent of i (and |S|)
  - Constant cost, "O(1)"

```
S: array_of_names;
n := |S|;
for i = 1..n-1 do
  for j = i+1..n do
    if S[i]>S[j] then
      tmp := S[i];
      S[i] := S[j];
      S[j] := tmp;
    end if;
  end for;
end for;
```

- Let's use a linked list for storing S
  - Create a class C holding a string and a pointer to an object of C
  - Put first s∈S into first object and point to second object, put second s into second object and point to third object, …
  - Keep a pointer p_0 to the first object

Selection Sort with Linked Lists

```
1.  i := p0;
2.  repeat
3.    j := i.next;
4.    repeat
5.      if i.val > j.val then
6.        tmp := i.val;
7.        i.val := j.val;
8.        j.val := tmp;
9.      end if;
10.     j = j.next;
11.   unil j.next = null;
12.   i := i.next;
13. until i.next = null;
```

- How much do the algorithm's steps cost now?
  - Assume following a pointer costs c
  1. One assignment
  2. Nothing
  3. One assignment, n-1 times
  4. Nothing
  5. One comparison, … times
  6. …
- Apparently no change in complexity
  - Why? Only sequential access

Example Continued

```
i := p0;
repeat
  j := i.next;
  repeat
    if i.val > j.val then
      tmp := i.val;
      i.val := j.val;
      j.val := tmp;
    end if;
    j = j.next;
  unil j.next = null;
  i := i.next;
until i.next = null;
```

- No change in complexity, but
  - Previously, we accessed array elements, performed additions of integers and comparisons of strings, and assigned values to integers
  - Now, we assign pointers, follow pointers, compare strings and follow pointers again
- These differences are not reflected in our “cost model”, but may have a big impact in practice

Content of this Lecture

- This lecture
- Algorithms and Data Structures
- Concluding Remarks

Why do you need this?

- You will learn things you will need a lot through all of your professional life
- Searching, sorting, hashing – cannot Java do this for us?
  - Java libraries contain efficient implementations for most of the (basic) problems we will discuss
  - But: Choose the right algorithm / data structure for your problem
    - TreeMap? HashMap? Set? Map? Array? …
    - “Right” means: Most efficient (space and time) for the expected operations: Many inserts? Many searches? Biased searches? …
- Few of you will design new algorithms, but all of you often will need to decide which algorithm to use when
- To prevent problems like the ones we have seen earlier

Exemplary Questions

- Give a definition of the concept “algorithm”
- What different types of complexity exist?
- Given the following algorithm …, analyze its worst-case time complexity
- The following algorithm … uses a double-linked list as basic set data structure. Replace this with an array
- When do we say an algorithm is optimal for a given problem?
- How does the complexity of an algorithm depend on (a) the data structures it uses and (b) the complexity of the problem it solves?